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Anna PAWŁOWICZ
WSK „PZL Rzeszow" SA
Adam MARCINIEC
Rzeszow University of Technology

A COMPUTATIONAL APPROACH TO THE BENDING AND CONTACT STRESS ANALYSIS OF SPUR GEARS

This paper presents the utilization of a three-dimensional (3D), finite element method to conduct root bending stress and surface contact stress calculations of a pair of spur gears. Firstly a pair of parallel spur gears without errors and tooth modifications is defined in a CAD system. Then, FEM analysis is conducted. Using FEM to calculate the RBS the load is applied as a force to the tip of the gear, it is modeled as a linear force uniformly distributed along the face width and perpendicular to the tooth surface. The SCS is considered as nonlinear analysis, where the contact of the pair of gears is assumed on teeth flanks. The results are compared with the ISO standards, Lewis formula and Hertz equation. It was found that the calculated results are comparable.

Keywords: spur gear, FEM, contact stress, bending stress

Introduction

Gearing is one of the most effective methods of transmitting rotary motion and torque from one shaft to another. The design of an effective and reliable gearing system is governed amongst other factors by its ability to withstand high bending stress and surface contact stress experienced at the tooth root and at the contact flanges [5]. To overcome the difficulties and limitations associated with the use of this semi-empirical formula, currently used by designers, a number of authors have utilized the finite element method to predict the root bending stress and the surface contact stress in gears [1, 2, 7]. It is well known from the literature that the true bending stress at the tooth root of spur gears is different from the nominal values that are utilized from the calculation of load capacity, either by standard or the usual design rules. No problems arise in using a load capacity rating when the simplified values are compared with the results of the bending fatigue test whose limits are calculated with the same schematic method. The true bending stress at the tooth root however has different trends and values, and the designer must be aware of this difference, especially for light gears with narrow ribs and rims [2]. In this paper a comparison for two gears was undertaken. A drive gear is a full body model; a driven gear is thin rimmed. A contact problem is a non-linear issue. The results achieved are compared with Hertz equation, Lewis formulas and ISO standards. The geometrical parameters of the full-body and the thin-rimmed gears analysed in this paper are presented in Table 1.

Table 1. Parameters of gears
Tabela 1. Paremetry kół zębatych

	Drive gear (full-body)	**Driven gear (thin rimmed)**
Number of teeth	23	46
Tip diameter [mm]	150	288
Pitch diameter [mm]	138	276
Root /Rim diameter [mm]	122.8	260.8 / 237
Face width [mm]	50.8	50.8
Normal pressure angle [deg.]	20	20

The Lewis formula

The classic method of estimating the bending stresses in a gear tooth is the Lewis formula. It models the gear tooth taking the full load at its tip as a simple cantilever beam. The maximum bending stress is presented below.

$$\boxed{\sigma_F = \frac{F_t P}{bY}} \tag{1}$$

where:

$$Y = \frac{2xP}{3} \quad \text{and} \quad x = \frac{t^2}{4l}.$$

In above equation: b is the face width, P diametric pitch, Y Lewis form factor, F_t is the tangential load. The form factor, Y is a function of the number of teeth, pressure angle and involutes depth of the gear. Assuming that the maximum bending stress is at point “a” (Fig. 1). By similar triangles we get x. It accounts for the geometry of the tooth, but it does not include stress concentration [6]. When considering the Lewis stress model of the drive gear (full-body) and driven gear (thin rimmed) results for applied moments (600, 1200, 1600 Nm) are presented in Table 2.

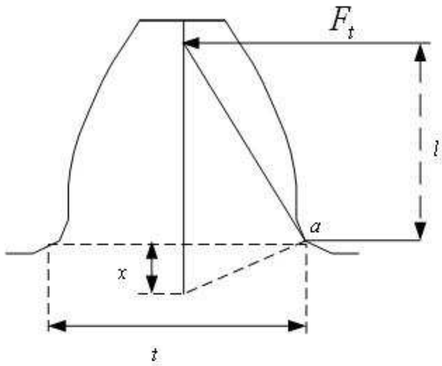

Fig. 1. Explanation of the Lewis form parameters
Rys. 1. Objaśnienie parametrów formuły Lewisa

Table 2. Results of Lewis formula
Tabela 2. Wyniki obliczeń wg formuły Lewisa

Moments applied [Nm]	**RBS calculated by Lewis formula**	
	drive gear [MPa]	**driven gear [MPa]**
600	100.74	72.26
1200	201.46	144.53
1600	268.63	192.70

The Hertz equation

Hertz contact between two cylinders is used for evaluating contact conditions at the major point of the contact for meshing gears. For computational determination stress at gear teeth flanks, Hertz theory is used. According to Hertz theory the distribution of surface contact stress in the contact area can be determined by:

$$\boxed{\boldsymbol{\sigma}_{\mathbf{H}} = \frac{4\mathbf{F}}{\mathbf{L}\pi\mathbf{B}}} \tag{2}$$

where:

$$\mathbf{B} = \sqrt{\frac{16\mathbf{F}(\mathbf{K}_1 + \mathbf{K}_2)\mathbf{R}_1\mathbf{R}_2}{\mathbf{L}(\mathbf{R}_1 + \mathbf{R}_2)}}, \quad \mathbf{K}_1 = \frac{1-\boldsymbol{\nu}_1^2}{\pi\mathbf{E}_1}, \quad \mathbf{K}_2 = \frac{1-\boldsymbol{\nu}_2^2}{\pi\mathbf{E}_2}.$$

Where F is the normal contact force, ν denotes a Poisson's ratio, E is the modulus of elasticity, L is the face width, $\mathbf{R}_1\mathbf{R}_2$ are radius of pinion and gear.

A Hertz equation is used to calculate the SCS. In the considered model the Hertz stress for applied moments are presented in Table 3.

Table 3. Results of Hertz equation
Tabela 3. Wyniki obliczeń wg równania Hertza

Moments applied [Nm]	**SCS calculated by Hertz equation [MPa]**
600	634.64
1200	898.52
1600	1036

ISO standards

The ISO published standards ISO 6336/1, 6336/2 and 6336/3 can be used to calculate the RBS and SCS of a pair of spur gears and helical gears. Equation (3) is the formula used by ISO 6336/3 for the RBS calculation. Equation (4) is the formula used by ISO 63363/2 for the SCS calculation for a pair of spur gears and helical gears having contact ratios in the range 1 < $\boldsymbol{\varepsilon}$ < 2. The meanings of the symbols used in equation (3) and (4) are described in Table 4 and 5. Since it is a very difficult thing to determine the values of $K_A, K_{V,} K_{F\beta,} K_{F\alpha}$ $K_{H\beta,} K_{H\alpha}$ exactly, the gears used here are regarded as ideal gears. The values of $K_A, K_{V,} K_{F\beta,} K_{F\alpha}$ $K_{H\beta,} K_{H\alpha}$ are equal 1. After the values of all the factors in Table 4 are given based on ISO standards, the RBS σ_F of the wheel can be calculated by substituting all the factors into equation (3).

$$\sigma_F = \frac{F_t}{bm_n} Y_{Fz} Y_{Sa} Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} = \frac{F_t}{bm_n} Y_{FS} Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} \tag{3}$$

The SCS $\boldsymbol{\sigma_H}$ of the gear may be calculated by substituting all factors into equation (4). All the factors are given in table 5 [3].

$$\sigma_H = Z_D Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b} \frac{U+1}{U}} \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{4}$$

Table 4. Meanings of the symbols used in equation (3), for applied moments: 600, 1200, 1600 Nm
Tabela 4. Wartości parametrów równania (3) dla momentu: 600, 1200, 1600 Nm

		Drive gear (full-body)	**Driven gear (thin rimmed)**
Nominal tangential load [N] for moments: 600 Nm 1200 Nm 1600 Nm	F_t	8 695.65 17 391.30 23 188.41	8 695.65 17 391.30 23 188.41
Face width [mm]	b	50.8	50.8
Normal module [mm]	m_n	6	6
Application factor	K_A	1	1
Dynamic factor	K_V	1	1
Face load factor	$K_{F\beta}$	1	1
Transverse load factor	$K_{F\alpha}$	1	1
Form factor	$K_{F\beta}$	1.99	1.41
Stress correction factor	$K_{F\beta}$	2.04	2.12
Contact ratio factor	Y_{Fa}	0.89	0.87
Helix factor	Y_b	1	1
Tip factor, equal to ($Y_{FS} \cdot Y_{Sa}$)	Y_{FS}	4.06	3.00
Tooth-root stress [MPa] for moment: 600 Nm 1200 Nm 1600 Nm	σ_F	103.00 206.01 274.68	74.48 148.97 198.63

The FEM analysis conducted to estimate root bending stress

A hexahedron solid element, which has eight nodes at the corners, is used. Applied force is modelled as a linear force uniformly distributed along the face width and tangential to the tooth surface applied to the tip. The model is fixed on the shaft. The drive gear has 17600 elements, the driven gear has 18318 elements type SOLID185. The analysis was carried out for three teeth of the gears considered, which improves efficiency and reduces the time needed for computation. The mesh was applied in a Catia v5 system. The material properties, boundary conditions and the applied moments were defined and calculated in Ansys software. The pair of spur gears were made from alloy steel. The following material data were used: E = 204.78 GPa (Young module) v = 0.26 (Poisson ratio). For defining the boundary condition a local cylindrical coordinate system for both gears was properly defined. For the drive and driven gear the average 1st Principle stress for the RBS achieved from the model are presented in Table 6. In the layout of a full body gear (drive gear) two maximum points located close to edges can be noticed. In the layout of a thin rimmed gear (driven gear) stress has one maximum located almost in the centre of the face width.

Table 5. Explanations of symbols used in equation (4)
Tabela 5. Objaśnienie symboli równania (4)

Moment applied	-	600 Nm	1200 Nm	1600 Nm
Nominal tangential load [N]	F_t	8695.65	17391.3	23188.41
Single pair tooth contact factor of the wheel	Z_D	1	1	1
Zone factor	Z_H	2.5	2.5	2.5
Elasticity factor	Z_E	186.96	186.96	186.96
Contact ratio factor	Z_ε	0.9704	0.9704	0.9704
Helix angle factor	Z_β	1	1	1
Reference diameter [mm]	d_1	138	138	138
Face width (mm)	b	50.8	50.8	50.8
Gear ratio (Z2/Z1)	U	2	2	2
Application factor	K_A	1	1	1
Dynamic factor	K_V	1	1	1
Face load factor	$K_{H\beta}$	1	1	1
Transverse load factor	$K_{H\alpha}$	1	1	1
Contact stress [MPa]	σ_H	618.71	874.99	1010.35

Table 6. Results of RBS calculated by FEM
Tabela 6. Wyniki obliczeń MES naprężeń gnących

Moments applied [Nm]	**RBS calculated by FEM [MPa]**	
	drive gear	**driven gear**
600	108.52	78.2
1200	217.44	156.68
1600	290.05	209.05

The FEM analysis conducted to estimate contact stress

For making the FEM model to calculate the SCS a hexahedron solid element, which has eight nodes at the corners, was used (drive gear has 90472 elements, driven gear has 84448 elements in Ansys named SOLID185, for contact element 170 is used for target 174). The applied moment is equal to: 600, 1200 and 1600 Nm [6]. The analysis was conducted for three teeth of the gears considered. The drive gear has axial and radial movement equal to 0, and rotational movement is allowed. The driven gear is clamped: $U_Z = U_R = U_\theta = 0$. Moment is applied to the node placed in axis of the drive gear. The node belongs to element Mass21, which is linked to the drive gear's edges by a stiff beam (element MPC184). FEM model is presented in Figure 2. The analysis does not include friction [4]. Contact was introduced on the flanks by surface-to-surface contact elements. Contact stress analysis was conducted for three loads (600, 1200 and 1600 Nm). The results of contact stress achieved by the FEM are presented in Table 7. Line contact is located on pitch diameter.

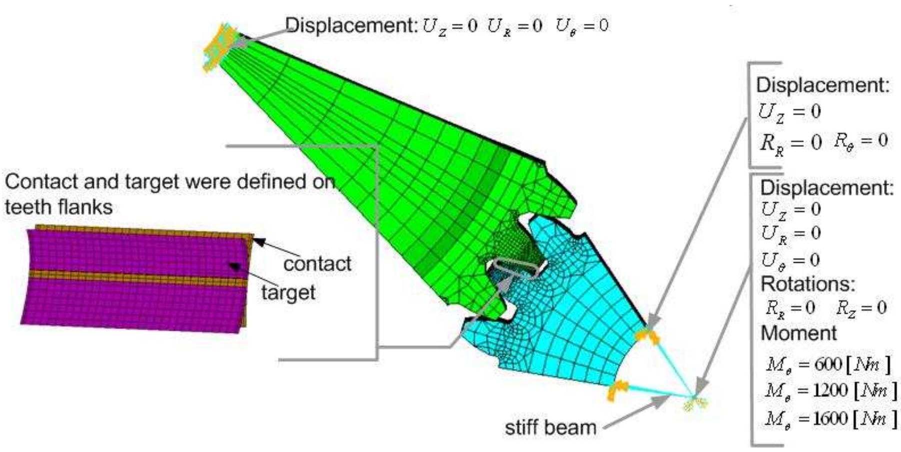

Fig. 2. Boundary condition applied
Rys. 2. Zastosowane warunki brzegowe

Table 7. Results of SCS calculated by FEM
Tabela 7. Wyniki obliczeń MES nacisków powierzchniowych

Moments applied [Nm]	**SCS calculated by FEM [MPa]**
600	715.69
1200	1024
1600	1174

Results and discussion

Comparing the results of the RBS it is noticeable that the Lewis equation can be used for a quick estimation of the stress on root diameter. The results are approximately 3% below the results achieved by the ISO standard. What is more, analyzing by the FEM value for the 1st principle average stress seems to be close to the ISO standard and the Lewis equation. The analysis was conducted for simple bending. The FEM results are approximately 5% above the results achieved in the ISO standard (Tab. 8). A different layout of stress for the RBS for thin and full body gears can be observed. A thin rimmed gear has one maximum of the RBS placed almost in the middle of its face width. For a full body gear two max stresses of RGB can be observed close to the edges.

Table 8. Results summary of RBS

Tabela 8. Podsumowanie wyników obliczeń naprężeń gnących

Moment applied	**Drive gear**					
	600 [Nm]	**deviation [%]**	**1200 [Nm]**	**deviation [%]**	**1600 [Nm]**	**deviation [%]**
ISO [MPa]	103.00	0	206.01	0	274.68	0
Lewis formula [MPa]	100.74	-2.2	201.47	-2.2	268.57	-2.2
1st Principle stress from FEM [MPa]	108.52	5.4	217.44	5.5	290.05	5.6

Moment applied	**Driven gear**					
	600 [Nm]	**deviation [%]**	**1200 [Nm]**	**deviation [%]**	**1600 [Nm]**	**deviation [%]**
ISO [MPa]	74.48	0	148.97	0	198.63	0
Lewis formula [MPa]	72.26	-3.0	144.53	-3.0	193.7	-2.5
1st Principle stress from FEM [MPa]	78.20	5.0	156.68	5.2	209.00	5.2

This article presents a way of conducting the SCS analysis for spur teeth by using the FEM method. The results achieved of the SCS are correct according to the ISO standard and Hertz equation (Tab. 9). The summary of results is presented in. If an assumption that the results calculated from the Hertz equation are 100% correct it can be noticed that results achieved by FEM contain approximately 14% error.

Table 9. Results summary for the SCS

Tabela 9. Podsumowanie wyników obliczeń nacisków powierzchniowych

Moment applied	600 Nm		1200 Nm		1600 Nm	
Hertz equation [MPa]	634.64	100%	898.52	100%	1036.37	100%
ISO standard [MPa]	618.71	3%	874.99	3%	1010.35	3%
FEM contact stress [MPa]	715.69	13%	1024.00	14%	1174.00	13%

Conclusions

This article presents a way of conducting the FEM model. The results achieved by FEM are acceptable, when compared to results with a well known method of SCS calculations (Hertz, and ISO) and RBS (Lewis, and ISO). In recent years, lightweight and compact design size for gears has been required, so strictly that gear designers must predict the real SCS and RBS of a pair of gears.

Based on it, the conclusion is that very similar error for different types of gears (e.g. spiral, hypoid) can be achieved. The only difficulty will be to prepare in CAD system a gear which will meet all geometric parameters. The different types of gears the SCS and RBS will be considered by the authors in the future.

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MODELOWANIE NUMERYCZNE NAPRĘŻEŃ GNĄCYCH ORAZ KONTAKTOWYCH W PRZEKŁADNIACH ZĘBATYCH

W pracy przedstawiono zastosowanie metody elementów skończonych w systemie 3D do obliczeń naprężeń gnących oraz powierzchniowych nacisków kontaktowych dla pary kół zębatych. Para kół zębatych, bez uwzględniania błędów kształtu oraz korekcji, została zdefiniowana w systemie CAD. Następnie przeprowadzono analizę z zastosowaniem MES. Obciążenie kół zamodelowano poprzez siłę działającą prostopadle do powierzchni zęba, rozłożoną równomiernie. Wyniki obliczeń numerycznych porównano ze standardami ISO, z obliczeniami zgodnie formułą Lewisa oraz równaniem Hertza – stwierdzono, że zyskane wyniki są porównywalne.

Słowa kluczowe: przekładnia zębata, MES, naciski powierzchniowe, naprężenia gnące

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